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Data and Orbit Analysis in Support of the U.S. Navy Satellite Doppler System

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Data and orbit analysis in support of the U.S. Navy satellite Doppler system

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This paper briefly describes the data and orbit analysis activities at The Johns Hopkins University Applied Physics Laboratory which support the U.S. Navy satellite Doppler system Tranet. Two topics are discussed in detail which are directly related to geodetic research, the major application of this system. These topics are the editing and archiving of the Doppler data and the generating of orbital ephemerides to an accuracy compatible with the 10 m accuracy of the experimental Doppler data.

1. Introduction—general activities

The United States Navy satellite Doppler system Tranet (R. R. Newton, this volume, p. 50) is maintained to provide accurate radio Doppler data and telemetry data for artificial near-Earth satellites. Technical responsibility for this system has been placed by the U.S. Navy at the Applied Physics Laboratory of The Johns Hopkins University. Analysis of the data from this system has been carried out by the Applied Physics Laboratory and the Naval Weapons Laboratory. This paper presents a brief description of the analysis activities at the Applied Physics Laboratory which support this system.

The existence of a satellite system implies considerably more than just a series of ground receiving stations collecting data. Figure 1 shows typical computation and analysis activities which support the Tranet system. From figure 1 it can be seen that many activities other than orbit computations are performed. These include continual analysis of the satellite performance such as satellite temperatures, status of the power supplies, command modes of the satellites, etc., as well as similar activities for the data receiving stations. Such activities result in what are labelled as diagnostics of the elements of the system. These diagnostics enable detection of any malfunctions or degraded performance of the various elements of the system and provide information to aid in taking immediate corrective actions by the system. One essential requirement for such monitoring activities is that they must be continual and close to 'real time'.

Scientific experiments in which the system is engaged require a more obvious support by analysis activities and are divided into two major categories in figure 1. Scientific telemetry analysis of 'on-board' experiments include preliminary reduction, editing, and archiving of such items as magnetic field measurements, radiation and particle count data, satellite attitude data, and performance of any experimental components of satellites or stations that may be undergoing study for inclusion into the system. The major use of the Tranet system has been for geodetic research. For this reason the major analysis activity is the analysis and editing of the satellite Doppler data to provide archives of accurate range rate information on the satellite motion. Such analysis includes accurate updating of satellite orbits which can be used, of course, for all research and diagnostic activities and maintenance of archives of related geophysical phenomena. Such phenomena include magnetic

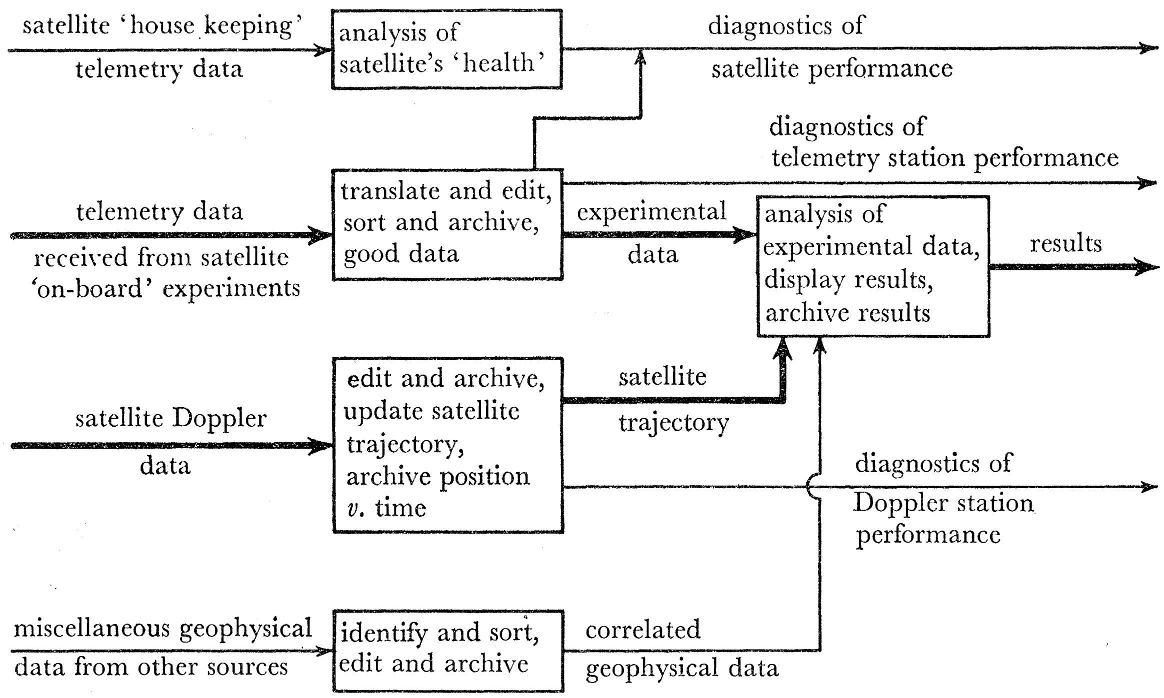


Figure 1. Analysis of orbiting satellite experiments.

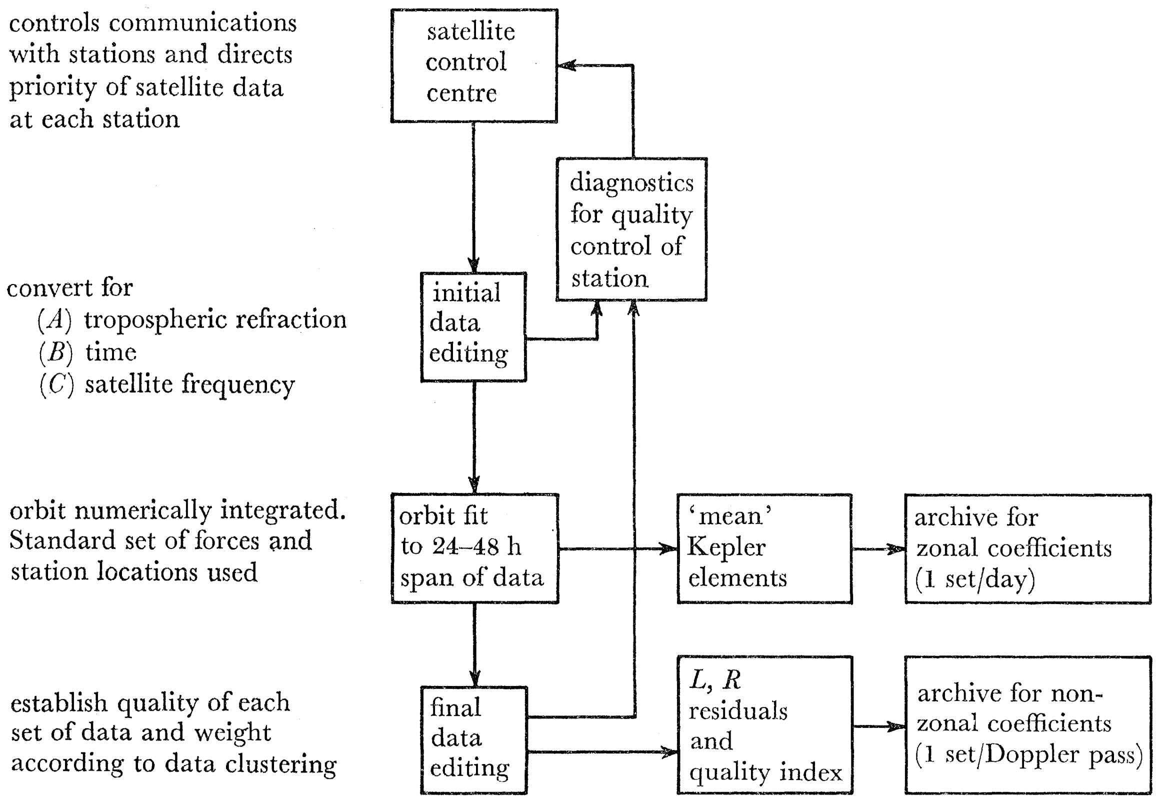


Figure 2. Preliminary processing and archiving of Doppler data.

and solar activity and motion of the Earth's spin axis relative to the mean geographic north pole.

Clearly it is impossible in one paper to adequately discuss all of these analysis activities. Consequently, I have limited detailed discussion to two topics closely related to geodetic research. These are the editing and archiving of the Doppler data and the generating of accurate satellite orbit parameters and ephemerides.

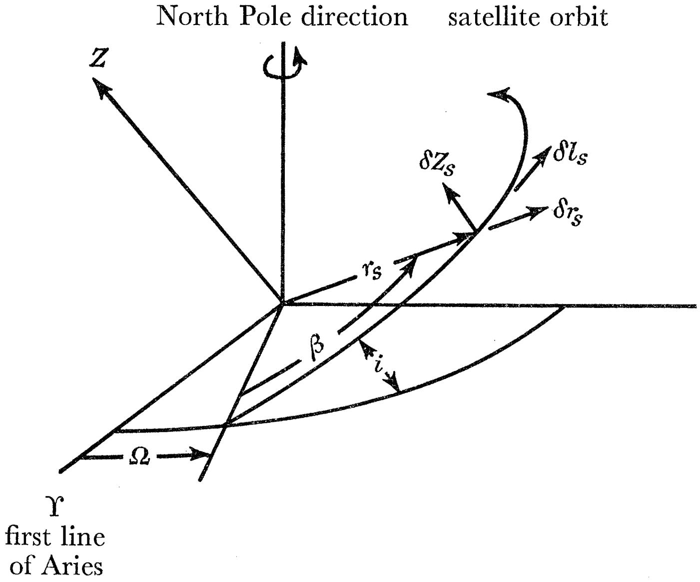
Figure 2 shows in more detail the analysis flow for archiving the Doppler data. The first topic I shall discuss in detail is the data editing and generation of the quantities labelled as L and R residuals in figure 2. These residuals are archived for geodetic research in improving station locations and the geopotential (Guier & Newton 1965). The second topic is the generating of accurate satellite trajectories. Typically these trajectories span 24 to 48 h and are used in a variety of ways.

2. Editing and archiving Doppler data

In addition to the usual format and 'valid message' checks on the data received from the Doppler stations, a very careful data point by data point edit is made in which spurious or noisy data points are deleted. This data editing is not performed by the usual comparison of each data point with its neighbours via a local fit to a polynomial or local smoothing function because of the possibility that erroneous data can still be 'smooth'. The basis for the data point editing is a theorem which states that when the station location and satellite orbit are nearly correct, the time dependence of the relative geometry of a satellite pass can be accurately approximated by a two-parameter fit to the data. I shall first discuss this theorem and then show how it is used in the data editing.

Consider a single satellite passing above the horizon of a station in which the approximate position of the satellite and station are known as a function of time. We now consider differential corrections to these coordinates such that the relative geometry between the satellite and station is correctly maintained throughout the pass. Figure 3 shows the approximate orbit parameters and the differential orbit correction in radius, along-track, and cross-track. From figure 3 it can be seen that the differential along-track coordinate is normal to the satellite radius vector, in the direction of satellite motion, and is in the orbital plane. The cross-track coordinate is parallel to the angular momentum vector of the motion and normal to both the radial and along-track differential coordinates. Figure 3 presents an approximate expression for the differential coordinates in terms of small changes in the values of the osculating Kepler elements of the motion. This differential motion of the satellite during the time of the satellite pass, together with corresponding expressions for the differential trajectory of the station's motion in inertial space, can be used to derive a corresponding differential change in relative geometry between the satellite and station. Finally, this differential change in the motion of the satellite relative to the station can be used to express the time dependence of data residuals based on the initial approximate values for the satellite and station trajectories (Guier 1963).

Tables 1 and 2 show the result for Doppler data residuals. In table 1 the differential frequency off-set between the station's local oscillator and the satellite transmitter frequency is included and then the time dependence of the residuals are expanded into a series of orthogonal functions $\{u_{s,k}(t-t_c)\}$ being that set of orthogonal functions symmetric in time relative to the time of closest approach, t_c, and $\{u_{A,k}(t-t_c)\}$ the corresponding set of functions that are anti-symmetric in time. In Guier (1963) it is shown that a set of functions can be found such that to zero order the coefficients of the first symmetric and anti-symmetric functions are the position of the satellite relative to the station at the time of closest approach. The first symmetric and anti-symmetric coefficients are shown in table 2.



a	semi-major axis
ϵ	eccentricity
i	inclination
Ω	right ascension of ascending node
ω	argument of perigee
β	argument of latitude
δr_s	differential radial coord.
δl_s	differential along-track coord.
δZ_s	differential cross-track coord.

$$\begin{aligned}\delta r_s(t) &= \delta a - a[\delta\epsilon \cos(\beta-\omega) + (\epsilon\delta\omega)\sin(\beta-\omega)] + O(\epsilon\delta r_s)\\ \delta l_s(t) &= a[\delta\beta + \cos i\delta\Omega] + 2a[\delta\epsilon\sin(\beta-\omega) - (\epsilon\delta\omega)\cos(\beta-\omega)] + O(\epsilon\delta l_s)\\ \delta Z_s(t) &= a[\delta i \sin\beta - \delta\Omega \sin i\cos\beta] + O(\epsilon\delta Z_s)\end{aligned}$$

FIGURE 3. Differential satellite orbit.

TABLE 1. CONTRIBUTIONS TO DOPPLER RESIDUALS I: FUNCTIONAL FORM

$$\delta\Delta f_D(t) = \delta^2 f + L_0 u_{s,0}(t-t_c) + L_1 u_{s,1}(t-t_c) + \ldots + R_0 u_{A,0}(t-t_c) + R_1 u_{A,1}(t-t_c) + \ldots$$

$$\begin{aligned}\delta\Delta f_D(t) &= \text{Doppler data residual},\\ &= \Delta f_{D_{\text{th.}}}(t) - \Delta f_{D_{\text{expt.}}}(t),\end{aligned}$$

where t_c is the time of closest approach; $\delta^2 f$ the oscillator frequency residual; $L_0, L_1, \ldots,$ coefficients of symmetric contributions, $R_0, R_1, \ldots,$ coefficients of anti-symmetric contributions, $u_{s,k}(t) = u_{s,k}(-t)$ and $u_{A,k}(t) = -u_{A,k}(-t)$.

From table 2 it can be seen that the coefficient of the first symmetric function, L_0, is the position of the satellite relative to the station in the direction of the along-track differential satellite position at the time of closest approach. Also, the first anti-symmetric coefficient, R_0, is the relative position resolved along the direction of the slant range vector at the time of closest approach.

Such an expansion can be employed in two ways. First, if the expansion in such functions is rapidly convergent, the expansion provides a method for reducing the data residuals below their noise level by the proper adjustment of a very few parameters associated with the relative geometry at the time of closest approach of the pass. Secondly, an archiving of

TABLE 2. CONTRIBUTIONS TO DOPPLER RESIDUALS II: COEFFICIENTS

principal symmetric coefficient

$$L_0 = \delta l_s(t_c) - \delta l_{\text{sta.}} + O\left(\frac{\rho(t_c)}{r_s}\,\delta l_s\right),$$

$$L_1 = O\left(\frac{\rho^2(t_c)}{r_s^2}\,\delta l_s\right),$$

where $\delta l_{\text{sta.}}$ is the station position error in along-track direction at t_c.

principal anti-symmetric coefficient

$$R_0 = \delta r_s(t_c)\cos\theta - \delta Z_s(t_c)\sin\theta - \delta\rho_{\text{sta.}} + O\left(\frac{\rho(t_c)}{r_s}\,\delta r_s\right),$$

$$R_1 = O\left(\frac{\rho(t_c)}{r_s}\,\delta r_s\right),$$

where $\delta\rho_{\text{sta.}}$ is the station position error in slant range direction at t_c, θ the angle between slant range and satellite radius vectors, $\rho(t_c)$ the minimum slant range and r_s the satellite radius.

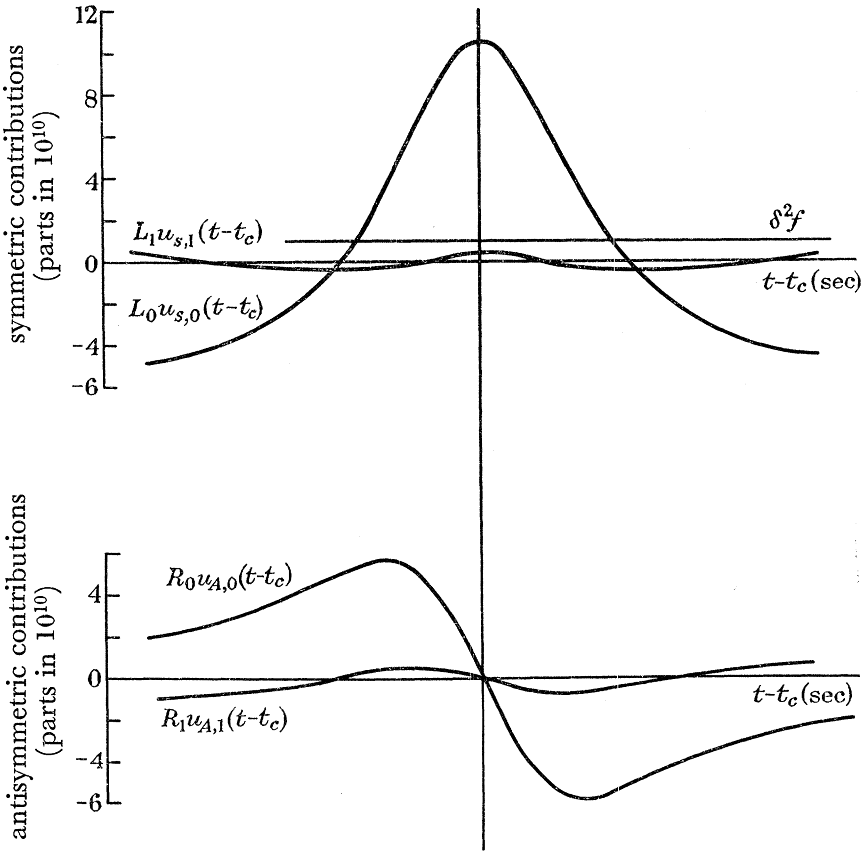


FIGURE 4. Contribution to Doppler residuals III: example. Satellite altitude 1 Mm; pass elevation, 45°; frequency error, $1/10^{10}$; position errors 100 m.

such L and R residuals provides an excellent set of aggregated parameters to use in geodetic research to improve satellite forces and station locations (Guier & Newton 1965). These expansion coefficients are the L and R residuals noted in figure 2.

Figure 4 shows a typical example of a satellite pass where the Doppler residuals have been reduced by adjusting the residuals for a frequency error, $\delta^2 f$, the relative along-track position error, $L_0 u_{s,0}(t-t_c)$, and the relative slant range position error $R_0 u_{A,0}(t-t_c)$. It can be seen that for position errors of 100 m, the Doppler residuals are reduced below 1 part in 10^{10} of the satellite transmitted frequency.

Figure 5 shows a schematic of the computer program which utilizes this ability to reduce the residuals to below the noise level as a method for deleting bad data (Black & Hook 1965). The computer algorithm can be seen to be one of least squares fitting the data residuals to the L and R coefficients, temporarily tagging for deletion any data point that

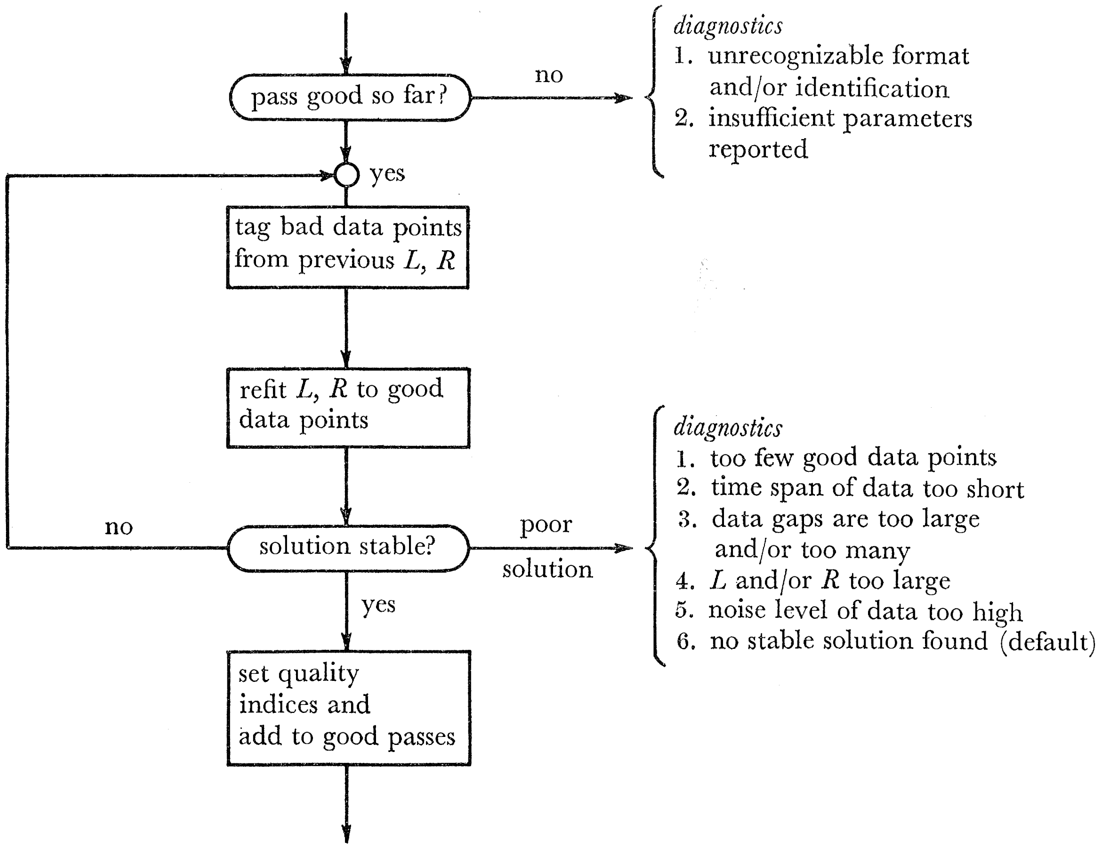


FIGURE 5. Data point edit for one pass.

deviates by a sufficiently large amount from the current value of the r.m.s. of the residuals. Assuming that the bad data points have been properly identified, the iterating of this process rapidly converges to a stable set of values for the L and R coefficients and those data points that are spurious. In practice it has been found that such a solution will stabilize within five iterations so long as there is less than about 30% bad data and will stabilize within three iterations for less than 10% bad data.

Also included in figure 5 are the diagnostics that may be printed describing the data if the pass is of poor quality. For data that are acceptable, 'quality indices' are associated with the data based upon similar criteria and are combined with the good data for all passes including the final values for the L and R parameters.

Figure 6 shows an example of an actual pass prior to and after data point editing. It can

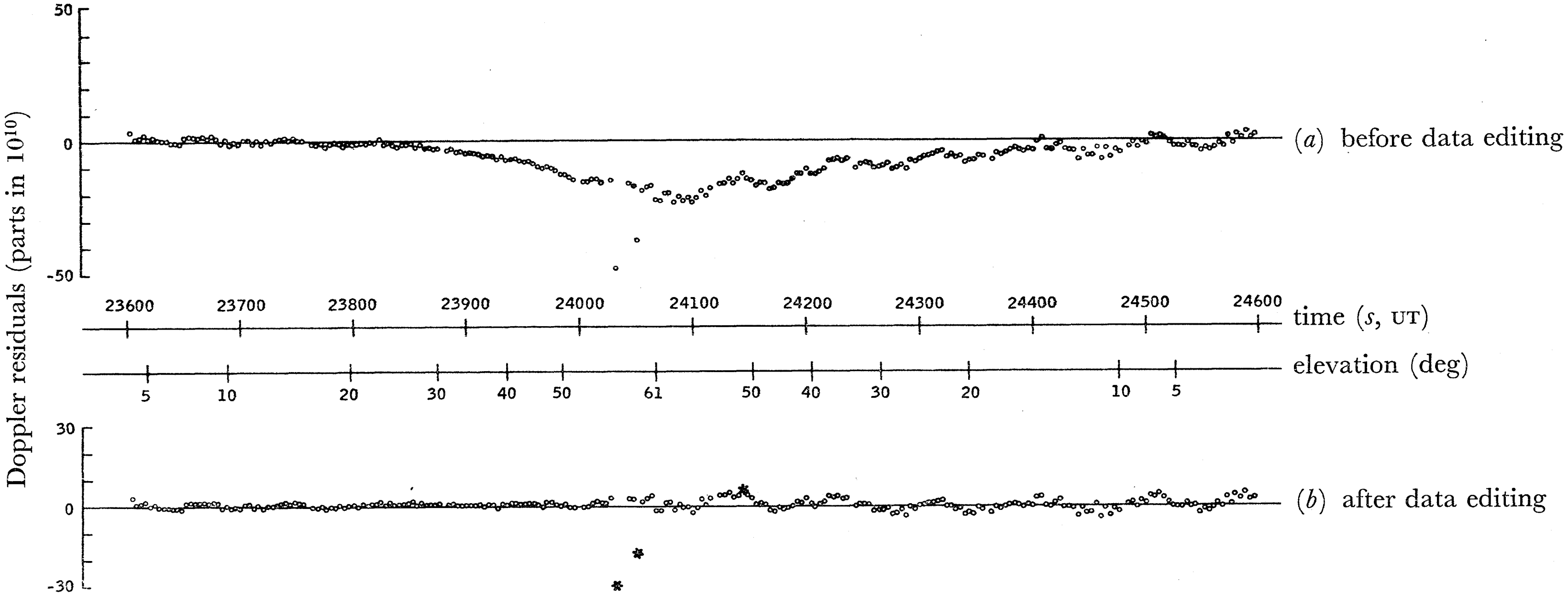


FIGURE 6. Doppler data point residuals. Satellite 1965 48C; 3 January 1966; station 111. $L_0 = -97$ m; $R_0 = +31$ m; r.m.s. $= 1{\cdot}7/10^{10}$; ○, accepted data; *, deleted data (error too large); data deleted below 5°.

be seen from this figure that a fit of only one symmetric and anti-symmetric coefficient are sufficient to reduce the residuals to the noise level. In doing so, three data points were deleted for producing too large a residual, even though they were less than 20 parts in 10^{10} of the transmitter frequency. Note also that data points corresponding to the satellite less than 5° above the station's horizon have been *ad hoc* deleted because of the unreliability of the tropospheric refraction correction of the data at these low elevations. This example was chosen especially because of the relatively large values of the coefficients (error in the satellite or station positions), typical values being 10 to 30 m. It can be seen that 100 m errors still effectively centre the residuals.

Table 3. Differential satellite orbit: change of initial values

$$t_0 = \text{epoch},$$
$$\delta\beta = \delta\beta_0 - \tfrac{3}{2}(\delta a_0/a_0)(\beta-\beta_0) + O(\epsilon).$$
$$\delta l_0 = a_0[\delta\beta_0 + \cos i_0\,\delta\Omega_0]; \quad \delta l_1 = -\tfrac{3}{2}\delta a_0;$$
$$\delta l_2 = 2a_0[\delta\epsilon_0 \sin\omega_0 + (\epsilon_0\delta\omega_0)\cos\omega_0];$$
$$\delta l_3 = 2a_0[\delta\epsilon_0 \cos\omega_0 - (\epsilon_0\delta\omega_0)\sin\omega_0].$$
$$\delta Z_1 = -a_0 \sin i_0\,\delta\Omega_0; \quad \delta Z_2 = a_0\,\delta i_0;$$
$$\delta r_s(t) = -\tfrac{2}{3}\delta l_1 - \tfrac{1}{2}\delta l_2 \cos\beta + \tfrac{1}{2}\delta l_3 \sin\beta + O(\epsilon\delta r_s),$$
$$\delta l_s(t) = \delta l_0 + \delta l_1(\beta-\beta_0) + \delta l_2\cos\beta + \delta l_3 \sin\beta + O(\epsilon\delta l_s),$$
$$\delta Z_s(t) = \delta Z_1 \cos\beta + \delta Z_2 \sin\beta + O(\epsilon\delta Z_s).$$

Once the data points within each satellite pass have been analysed in this manner a final editing of all the passes for the satellite arc in question is made to delete any passes that appear to be 'out of character' with respect to the other passes in the arc. Such an editing will detect, for example, an error in the station's clock. Clearly, if the satellite orbit has very recently been updated so that it is accurate, a simple examination of the magnitude of the L and R coefficients can be made. On the other hand, typically the orbit may be a few days old and consequently before the detection of the bad passes the L and R residuals must be altered to reflect an orbit improvement. Figure 9 shows a method for doing this by using the differential orbit parameter representation shown in figure 3. Letting the differential Kepler elements of figure 3 be constants provides an approximate description of the time dependence in the radial, along-track, and cross-track coordinates of the satellite, shown now in table 3. Evaluating these functions at the time of closest approach for each pass and least squares fitting the L and R coefficients with respect to the differential Kepler elements provides new least squares values for the L and R coefficients which can now validly be examined for bad passes.

Figure 7 shows the computer algorithm for deleting bad passes which is analogous to deleting bad points within a single pass. Again a few iterations of this loop typically will produce a stable solution for the differential orbit parameters and those passes which are not of sufficient quality to be used. Figure 10 also shows some typical diagnostics for bad passes. The resulting good passes together with their L and R residuals and quality indices are now archived for use in improving (updating) the satellite orbit and/or for geodetic research.

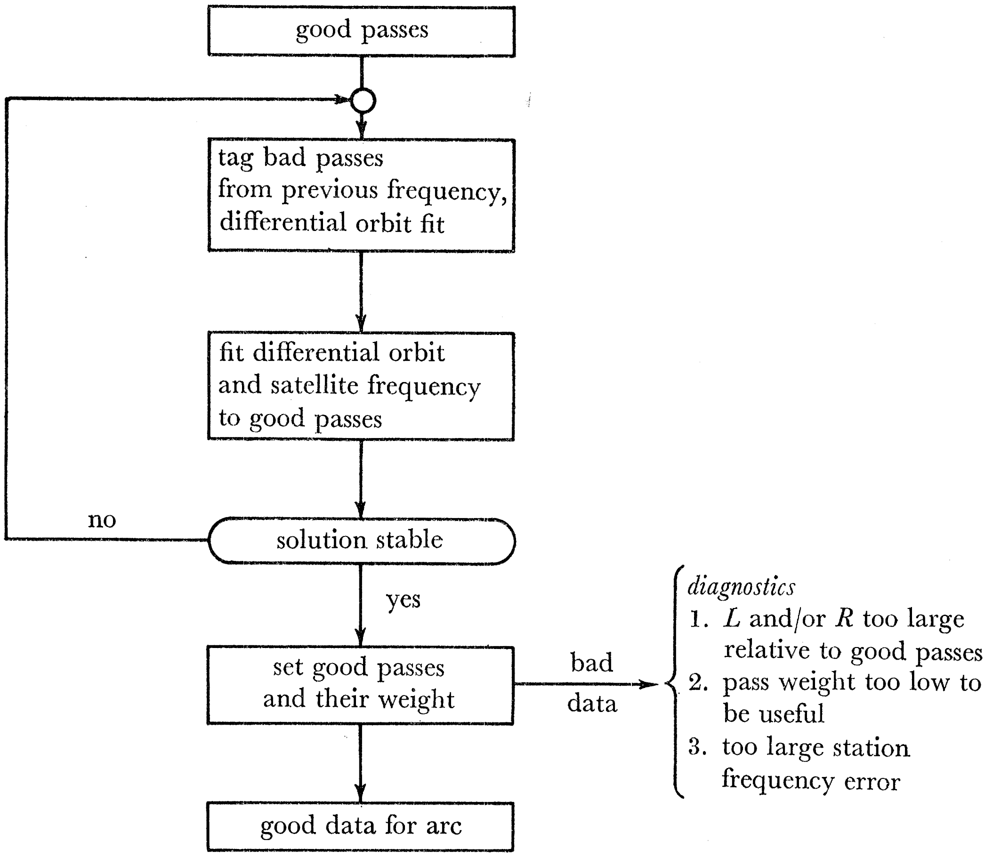


FIGURE 7. Pass edit for satellite arc.

3. GENERATING SATELLITE EPHEMERIDES—THE INTEGRATOR

The current observational accuracy of the Tranet system is believed to be about 10 m for the data from a single pass of a near-Earth satellite. Consequently to properly support the system the error with which satellite ephemerides are generated should be not more than about 10 m for arcs spanning 1 to 2 days. Such high accuracy requirements dictate that a numerical integration scheme must be used because of the detailed manner in which air drag, radiation pressure, gravity forces, etc., must be considered. This section describes the basic method that is used to numerically integrate orbits for support of the Tranet system and for geodetic research.

Two elements of numerical integrators control their accuracy. The first is the choice of orbit parameters where the parameters should be chosen such that there is a minimum variation in time of the parameters. The second is the choice of the numerical algorithm which approximates differential equations by difference equations. Figure 11 indicates the basic orbit parameters which have been chosen (Newton 1961). The second part of table 4 indicates the definition of these parameters in terms of the usual Kepler elements and consequently defines the scaling (units) of the parameters. With the semi-major axis scaled by the mean radius of the Earth the magnitude of the P vector is about unity. The magnitude of the e vector is the orbit osculating eccentricity. The independent variable has been chosen to be the argument of the latitude rather than time because the largest

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forces acting on the satellite are functions of its position with only a weak explicit dependence upon time. Consequently, a seventh variable is numerically integrated along with the three components of the P and e vectors. However, there are only six independent parameters since, by definition, the P and e vectors are always orthogonal.

Table 4. Numerical integrator I: orbit parameters

β	argument of latitude (independent variable)
$t(\beta)$	time satellite reaches value of β
$\mathbf{P}(\beta)$	angular momentum vector (normal to orbital plane)
$\mathbf{e}(\beta)$	'eccentricity' vector (direction points to perigee)
$\mathbf{P}.\mathbf{e} = 0$	

$$\mathbf{P}^2 = a(1-\epsilon^2)$$

$$\mathbf{P}_1 = P \sin i \sin \Omega, \quad P_2 = -P \sin i \cos \Omega, \quad P_3 = P \cos i$$

$$\mathbf{e}^2 = \epsilon^2$$

$$\mathbf{e}_1 = e(\cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i)$$

$$\mathbf{e}_2 = e(\cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i)$$

$$\mathbf{e}_3 = e \sin \omega \sin i$$

Table 5. Numerical integrator II: change in orbit parameters

$t_0(\beta)$	time unperturbed satellite reaches value of β
$\mathbf{P}_0 = \mathbf{P}(\beta = 0)$	angular momentum vector of unperturbed orbit
$\mathbf{e}_0 = \mathbf{e}(\beta = 0)$	eccentricity vector of unperturbed orbit

$$\Delta t(\beta) = t(\beta) - t_0(\beta)$$

$$\Delta\mathbf{P}(\beta) = \mathbf{P}(\beta) - \mathbf{P}_0 = \mathbf{P}(\beta) - \mathbf{P}(0)$$

$$\mathbf{e}(\beta) = \mathbf{e}(\beta) - \mathbf{e}_0 = \mathbf{e}(\beta) - \mathbf{e}(0)$$

$$P^2(\beta) = P_0^2 + 2\mathbf{P}_0 . \Delta\mathbf{P} + \Delta\mathbf{P} . \Delta\mathbf{P} \text{ (double precession)}$$

$$t(\beta) = t_0(\beta) + \Delta t(\beta) \quad \text{for } n\text{th revolution after epoch,}$$

$$t_0(\beta) = n[t_0(2\pi) - t_0(O)] + [t_0(\beta - 2\pi n) - t_0(2\pi n)] + t_0(0), \text{ (double precession)}$$

For an unperturbed orbit the variables in table 4 are constant. Consequently the fixed word length of a computer can be better used by transforming to the changes (perturbations) in these parameters due to perturbing satellite forces. The difference parameters are shown in table 5. After integration, the actual orbit parameters and time are constructed as shown in table 5. The most sensitive parameters to the accuracy of computing satellite position are the magnitude of the P vector and the time. These are carefully computed separately from the components of the two vectors. Consequently, in reality, eight parameters are numerically integrated, only a subset of six being independent, and with two being carried in double precession in the computer. Finally we note that, since the nodal period of the unperturbed orbit is constant, the unperturbed time can be carefully computed as indicated in the last equation in table 5.

The numerical algorithm for approximating the differential equations of motion for the $\Delta\mathbf{P}(\beta)$ and $\Delta\mathbf{e}(\beta)$ vectors and the time, $\Delta t(\beta)$, is Runge-Kutta, 4th order. Higher order could have been used but would have required more computer time and, with these variables, it has been found that the resulting accuracy is sufficient for present applications. Considering the rapid variation of the forces, particularly the radiation pressure and resonance geopotential coefficients, 64 steps per satellite revolution in the independent variable, β, is used.

Table 6. Orbit integration capability

accuracy

100 revolutions forward, 100 revolutions backwards in time (12800 integration steps)

$$|\Delta \mathbf{P}/P| = 8{\cdot}1\times 10^{-8}$$
$$|\Delta \mathbf{e}/e| = 1{\cdot}6\times 10^{-6} \quad (\epsilon = 0{\cdot}048)$$
$$\Delta t = 5{\cdot}1 \text{ ms}$$

100 days forward in time. (Actual satellite motion used as standard†) (85200 integration steps)

$$|\Delta \mathbf{P}/P| = 1{\cdot}3\times 10^{-5}$$
$$|\Delta \mathbf{e}/e| = 1{\cdot}0\times 10^{-3} \quad (\epsilon = 0{\cdot}033)$$

computer speed

5·1 min/day on IBM 7094-1

† Corrected *a priori* for tidal effects, corrected *a posteriori* for air drag effects and time of node.

Table 6 presents the accuracy and speed characteristics of this integrator which has been implemented on the IBM 7094 computer. The error shown in table 6 for 100 revolutions corresponds to approximately 15 m in satellite position. This accuracy is still somewhat better than the physical accuracy with which satellite forces are known. The code for the computation of the satellite forces has been refined considerably to increase the integration speed. From table 6 it can be seen that it requires approximately 5 min of computer time per day of satellite motion on the IBM 7094.

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